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COMPLEMENTARY DIVISION.

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THE object of the following investigation is to extend the well known facility of dividing by 10, 100, or 1000 etc., as far as practicable, to 9, 99, 999, and other adjacent divisors. For illustration, let 996 be the divisor, where $996+4 = 1000$. It will be seen presently that the smaller complement or 4 can be used in place of the larger 996; and this has suggested the name of the method.

Introductory to the Rule, a first Example is selected, in which the carrying figures from one period to the next, are known to be 0. After separating the dividend into periods by the Rule, we simply bring down the first left hand period 023 or 23 into the quotient, *multiply it by the complement* 4, *and add the product* 23×4 *to the next period* 068, writing the sum 160 underneath; and so on. Under the last period, we have $160 \times 4 + 069 = 709$ the Remainder.

In Example 2, the carrying figures 1, 4, 2, 0 have been computed mentally in advance. We add each to its period, and proceed with the sum as in Example 1, only omitting the same 1, 4, 2 when reproduced, since each was added previously. The power of 10 nearest the divisor is 1000.

EXAMPLE 1.		EXAMPLE 2.	
Divisor 996	$023'\ 068'\ 069$ Dividend.	996	$328'\ 571'\ 953'\ 274$
Complem't 4	$+0\ \ +0\ \ +0$	4	$+1\ \ +4\ \ +2\ \ +0$
+	$23160\frac{709}{996}$ Quotient.	+	$329891519\frac{350}{996}$.

It should be observed in Example 2, that, after adding the 1, 4, 2, successively, the standard process, which gave these again with the quotient figures, is a full repetition of the mental process previously employed or abbreviated to find the same carrying figures 1, 4, 2.

Demonstration.—In Example 2, let the divisor 996 take the form of 1000—4. And let c denote a correction such that $328+c$ shall express the first three figures of the quotient. Multiplying divisor by quotient, or 1000—4 by $328+c$ and subtracting the product, the first period of the dividend evidently disappears. Also by making the total carrying figure $1 = c$, the terms $c—1$ disappear, leaving the remainder $571+312+4c$ or 887.

The next step including the sum of 953 and 3548, that is 4′501, requires that $c' = 4$; so that $501+4c'$ gives the 517 remainder. To this we bring down the next period 274, and proceed as before.

	996	328′571′953′274	$328 + c = 329$
	1000—4	c–1 312 –4c	
		887 9c3 274	$887 + c' = 891$
$c = 1$;		c'–3 548 –4c'	
$c' = 4$;		517 274	$517 + c'' = 519$
$c'' = 2$.		c''–2068–4c''	
		$329891519\frac{350}{996}$, Quotient.	

Equivalent process.	996	328	571	953	274	Dividend.
	4	8+1	87+4	7+2		Correct'n.
	+	329	891	519	$\frac{350}{996}$	Quotient.

A.	*B*, ment'y	*C.*	*D.*	*E.*	
8	32×4+57	8+1	329×4+571	87	etc.
32	= 1(85)	329	= (1)887	8	

Rule I. For Division near 100, or some other Power of 10.

Rule.—Under the Divisor, write its Complement, such that the Algebraic sum of the two shall equal the nearest integer power of 10. Also, commencing on the right, separate the Dividend into Periods of as many figures each as there are cyphers in the chosen power of 10. Then bring down the first one or two digits of the left hand period into the Correction line, and the rest of the period into the lower line of the quotient, as shown at *A*, or *E*.

When any period of the Quotient is thus incomplete, multiply its two left hand figures (or cyphers, if the period is deficient) by the Complement, and mentally add the product to the corresponding figures in the next period of the dividend, as shown at *B*. Write the carrying figure of the period so found, on the correction line; also add it to the preceding figure, and write the sum in the quotient, as shown at *C*.

As soon as each period in the Quotient line is completed, multiply it by the Complement as indicated in *D*, and, adding the product to the next period of the dividend, write the sum underneath in the form *E* or *A*, but omit the final carrying figure or correction, which has been added previously.

NOTE 1. Should this true carrying figure found from the full period in *D*, be 1 more than the deficient figure previously written in the correction line, erase and correct accordingly. And correct, when there is 1 to carry in adding to form the quotient.

NOTE 2. At the end, the last period of the Dividend which gives the Remainder, has the carrying figure or correction always 0, as shown in the demonstration.

NOTE 3. In the case of Divisors like 103, 104, 1006, which exceed the nearest power of 10, we multiply by the Complement regarded as negative, and (instead of adding) subtract the products and the correction or carrying figure of the period.

EXAMPLES.

997	86 512 743 295 Dividend.	991	328 571 953
3	6+0 0+3 2+0	9	28+3 0+5
+	86 773 062 $\frac{481}{997}$ Quotient.	+	331 555 $\frac{948}{991}$.

96	3 28 57 19 53 Dividend.	9995	314 1592 6535
4	3+00+25+13+1	5	4+0 2+2
+	3 42 26 24 $\frac{49}{96}$ Quotient.	+	314 3164 $\frac{2355}{9995}$.

93	46 09 16 12 57	999	123 456 789 Dividend
7	6+32+48+08+5	1	3+0 9+1
+	49 56 08 73 $\frac{68}{93}$	+	123 580 $\frac{369}{999}$ Quotient.

By Note 1, erase (5) (6) (7)

998) 23025850929 98) 434294482

For verification of the quotients so easily found in practice, we employ from Arithmetic, the proof by excess of 9's. By supposing the digits of the dividend to be all 9's, we learn that whenever the mental process of the Rule gives a correction that is equal to the Complement of the divisor, such correction is exact, being the maximum value: also that the correction is exact when its omitted right hand digit is less than the right hand digit of the Divisor.

In the next Examples, where the Complement is *subtractive*, it will be most convenient to extend the process of "borrowing 10 units and paying 1 ten", to borrowing 20, 30 or 40 etc units, and paying 2, 3 or 4 etc. tens, respectively. Thus, in the first Example below, for 872—67 × 5, we begin with 2—7 × 5 or 2—35 and, borrowing 40 units, say 42—35 = 7, remainder. Then paying 4 tens to the next product 6×5, it becomes 34. And 87—34 leaves 53, so that 537 is the whole remainder.

Divisor. 1005	67 872 531 624 Div.	103	23 02 58 50 93
Comp't. 5	7—0 7—361—4	3	3—16—13—27—2
—	67 534 857 $\frac{339}{1005}$ *Q*'t.	—	22 35 51 95 $\frac{08}{103}$.

105	9 81 28 26 32 80	108	18 74 19 75 81 40
5	9—06—28—31—32—4	8	8—18—39—37—63—3
—	9 34 55 48 88 $\frac{40}{105}$	—	17 35 36 81 30 $\frac{100}{108}$.

By Note 1, erase (5) (9) (7)

In working with pencil, the (5) and (9) of the last Example but one, can be erased, and the 4 and 8 substituted by Note 1. And in the last Example, where the mental process of the Rule at first gave 7 — 7, the next step required erasure and correction corresponding to 7—6 as above shown. At the end, or for 4 — 3×8 it was necessary to borrow 30 units and pay 3 tens, since this is conformable to the Rule. The method is particularly adapted to Discount according to the divisors 103, 104, and other rates of interest. The signs of the corrections + or — can often be understood and so omitted. For further Exercises:

104	31 41 59 26.	107	42 31 50 01 48.
4	1—1	7	2—3
—	3	—	

106) 6262048599. 102) 123456789.

RULE II. DIVISION BY 9.

Rule.—Cast out the 9's from the dividend in the usual way and write the excess or remainder in fractional form under the right hand of the dividend, *subtract the upper from the lower digit,* borrowing 10 when required, and paying 1 as in subtraction, and writing each difference on the left successively, to form the quotient.

Thus in the example below, we first add the digits 7+5+2+1+4+9+2+8, throwing out in progress, the 9's of the sum to get the final excess 2, which is written as $\frac{2}{9}$ under the 8. Subtracting 8 from 2 or rather 8 from 12 leaves 4 to be written on the left. Carrying 1 to the 2 over the 4, we subtract 3 from 4, which leaves 1 to be written as before, and so on.

Examples. *Proof.* Multiplication by 9 or 10—1.

9) 75214928 Dividend.
83572144$\frac{2}{9}$ Quotient.

83572140
—8357214
75214926 Product.
+2 Remainder.

9) 1497120548
166346727$\frac{5}{9}$.

9) 372946
$\frac{4}{9}$.

9) 765291835

RULE III. DIVISION BY 11.

Rule.—Commencing with units place, add the alternate digits of the dividend; from their sum subtract the sum of the other digits, adding or rejecting 11 or 22, etc., when required to make the subtraction possible, and write the excess, so found, in fractional form under the right of the divi'd. *Subtract each lower digit from the digit above it,* writing the difference on the left successively, to form the quotient.

In the annexed Example, $8+5+6+4+3=26$; and $3+2+1+8=14$; also $26 - 14 = 12$; from which, rejecting 11 leaves 1, the excess of 11's. Writing this remainder 1 as $\frac{1}{11}$ under the 3 and subtracting 1 from 3 gives 2 in the quotient; 2 from 8 gives 6; and so on.

Examples. *Proof.* Multiplication by 11 or 10+1.

$$\begin{array}{rl|r||rl} 11\ )\ 835261483 & \text{Dividend.} & 26 & 759328620 & \\ 75932862\tfrac{1}{11} & \text{Quotient.} & -14 & \underline{75932862} & \\ & & \overline{12} & 835261482 & \text{Product.} \\ & & 1 & +1 & \text{Remainder.} \end{array}$$

$$\begin{array}{r|l} 11\ )\ \underline{621345987321} & 22+11 \\ 56485998847\tfrac{4}{11} & \underline{-29} \\ & 4 \end{array} \qquad \begin{array}{r|l} 11\ )\ 8990757556 & 25+11 \\ 596\tfrac{0}{11} & \underline{-36} \\ & 0 \end{array}$$

RULE IV. DIVISION BY LARGE NUMBERS.

Rule.—Multiply both divisor and dividend, as if numerator and denominator, by any preparatory multiplier (as given in the annexed Table), such that the left hand part of the new divisor shall be near 99, or less than 100; and take its complement or difference from the nearest power of 10. *Then will the sum of the Dividend and sub-products give the Quotient figures successively.*

Thus, write the first left hand figure of the new Dividend (as 6) in the Quotient; multiply it by the Complement, and set the product under the Dividend, extending as many places to the right of the left figure (6) as there are digits in the whole Divisor.

Add the next figure of the Dividend (as 3) to any product figures vertically under it, including any carrying figure from the right. Enter this sum as the next figure of the Quotient, multiply it by the Complement, and set the product under the previous prod., but one place farther to the right.

Again, add the next figure of the Dividend (as 5) downward, proceeding as already described; and so on.

After three or four Quotient figures are thus found, it will be convenient to add all the remaining figures, whose sum will be the Remainder or a new Dividend, with which to proceed as before.

EXAMPLE.

```
Divisor              9876543 | 6437695      Quotient.
Complement            123457 | 6358217483   Dividend.
                           + |   740742         6×Complement.
        Continued.              493828      4     "
      ...338338  Quotient ⎫     370371      3     "
123457 | 3341615 Remain.  ⎪     600334          New Dividend.
  3833 |   37037          ⎪      864199     7×Complement.
            3704          ⎬       740742    6     "
             987          ⎪       1111113   9     "
Contraction.  3343        ⎪        617285   5     "
                37        ⎭       3341615   Remainder.
```

Here subtracting the right hand figure of the Divisor from 10, and the others from 9, we obtain the complement 123457. Multiplying this by 6, the left figure common to the Dividend and the Quotient, we write the first figure, 2 of the product, seven places to the right of the 6, since there are seven places in the whole Divisor. Then the next figure of the Dividend, 3, increased by 1, carrying figure, gives 4 in the quotient. Continuing the process as described in the Rule, we obtain 6437695 Quotient, and 3341615 Remainder.

Contraction.—As shown on the left, above, the process is continued by writing the next quotient figure, 3, both in the Quotient, and under 5, the second figure of the Complement, to apply contracted multiplication. And thus we obtain further 338338, making these thirteen places in the Quotient, 6437695338338.

By the the usual operation of Long Division, each quotient figure would cost the writing of sixteen other figures; so that 208 other figures would be required with the corresponding mental labor to determine the quotient 6437695338338; where the Complementary process properly requires only 78 figures, and the mental labor is much shorter and easier. In general, the more 9's on the left of the Divisor, the smaller the Complement and the easier the process of Division.

With respect to the preliminary Multiplier of the Rule, since the product of the Divisor by its reciprocal is always 1, the required Multiplier will be a little less than the reciprocal of the Divisor, and may thus be found from the published Tables of Reciprocals.

Less exact Multipliers are given in the following outline Table which may serve for illustration. The values can be easily verified by the common Table of Reciprocals.

PREPARATORY MULTIPLIER FOR RULE IV.

First Divisor	Multiplier.	First Divis.	Multiplier.	First Divis.	Multiplier.	First Divis.	Multiplier.	First Divis.	Multiplier.
100	9×11	141	7	201	7×7	304	4×8	589	4×4
104	8×12	150	6×11	205	6×8	323	3	626	3×5
110	9	154	8×8	218	5×9	345	4×7	667	2×7
113	8×11	157	7×9	223	4×11	358	3×9	779	12
118	7×12	164	6	233	6×7	385	$\frac{1}{4}$	834	11
122	9×9	176	7×8	244	4	401	4×6	869	$11\frac{1}{2}$
124	8	179	5×11	257	38	435	2×11	918	9×12
129	7×11	182	6×9	271	6×6	455	3×7	944	$10\frac{1}{2}$
132	$\frac{3}{4}$	189	$5\frac{1}{5}$	278	5×7	477	2	975	102
137	8×9	197	5	295	3×11	527	3×6	990	1

The following Divisor and Dividend are first multiplied by 8, since 124 in the Divisor indicates this Multiplier. The new Dividend is written one space lower to make room for the Quotient over it; and after the first addition, Contraction is applied. In the quotient, the more full carrying figure once changes 8 to 9: *Example:*

```
First Divisor, 124689236 ) 4791562834 First Dividend.
Multiplier,            8           8
Divisor,       997513888 | 38428039081 Quotient.
Complement,      2486112 | 38332502672 Dividend.
From Quo't,     18093082 |    7458336
                              19888896
                                9944448
                              27969372 8
                                  497222
                                  198889
                                     746
                                     224
                                     809
                                       2
```

```
Example           3521837 ) 8263541783×4
for Practice,    14087348 ) 33054167132×7
                 98611436 |               Quotient.
Complement,       1388564 | 231379169924  Dividend.
```

The Rule can also be applied to Divisors beginning with 100 or 101, by omitting the 100 or 10 on the left, and taking the remaining part as the negative Complement, whose products or the sum of products by quotient figures are to be subtracted from the Dividend.